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濃縮ガスによる低速中性子散乱を用いた未知短距離力探索の手法

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contents

1.Introduction

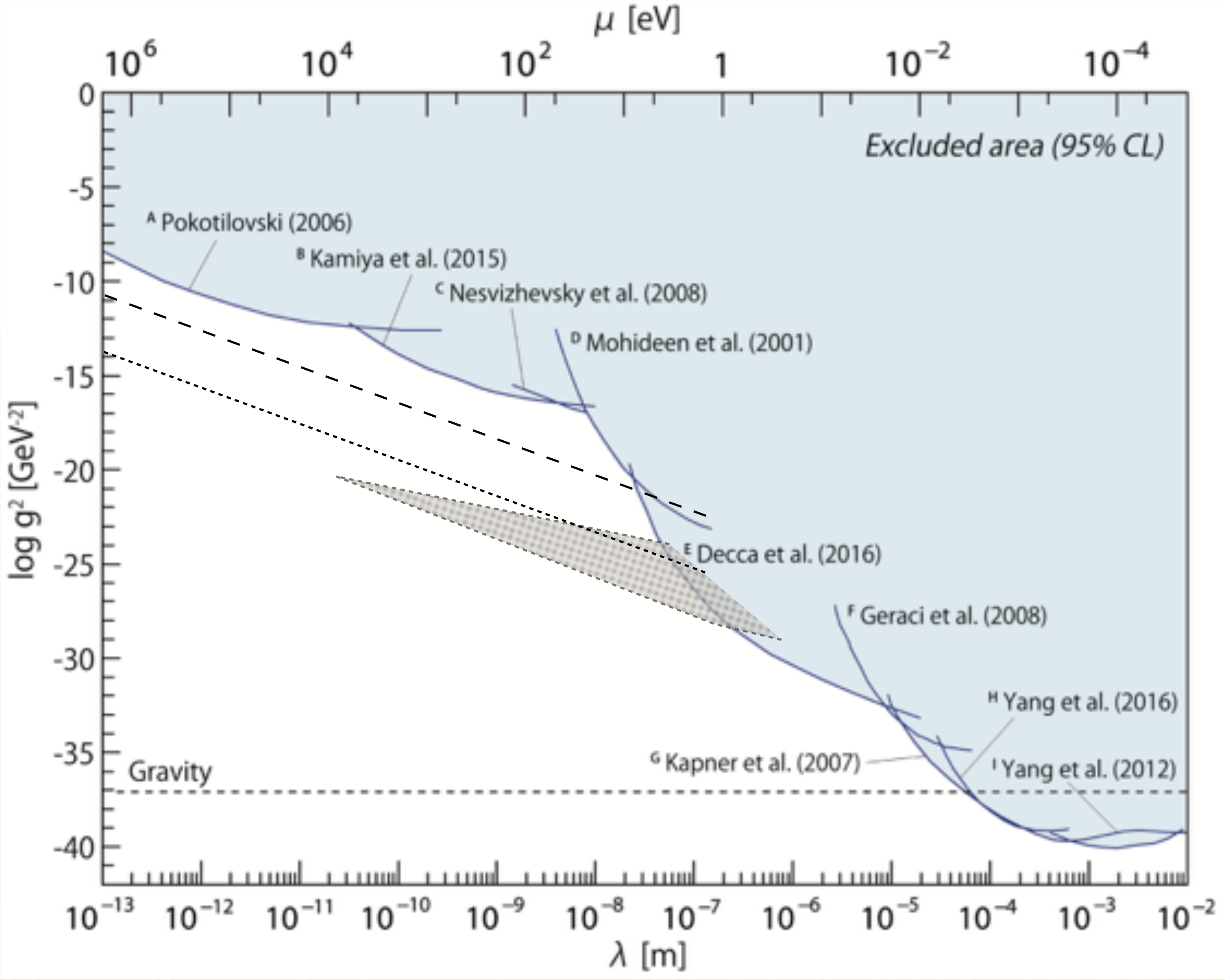
Yukawa-type new interactions

coupling strength

$$V_{\rm new}(r) = -\frac{1}{4\pi} g^2 Q_1 Q_2 \frac{e^{-\mu r}}{r}$$

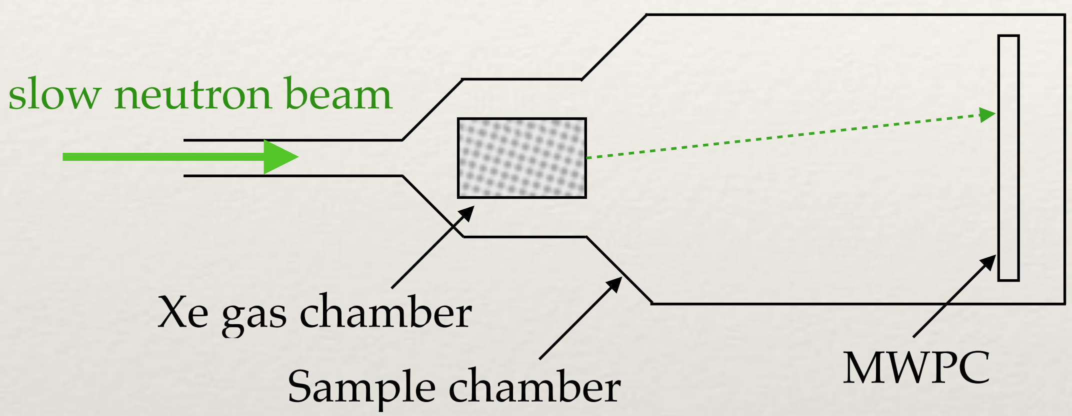
coupling charges

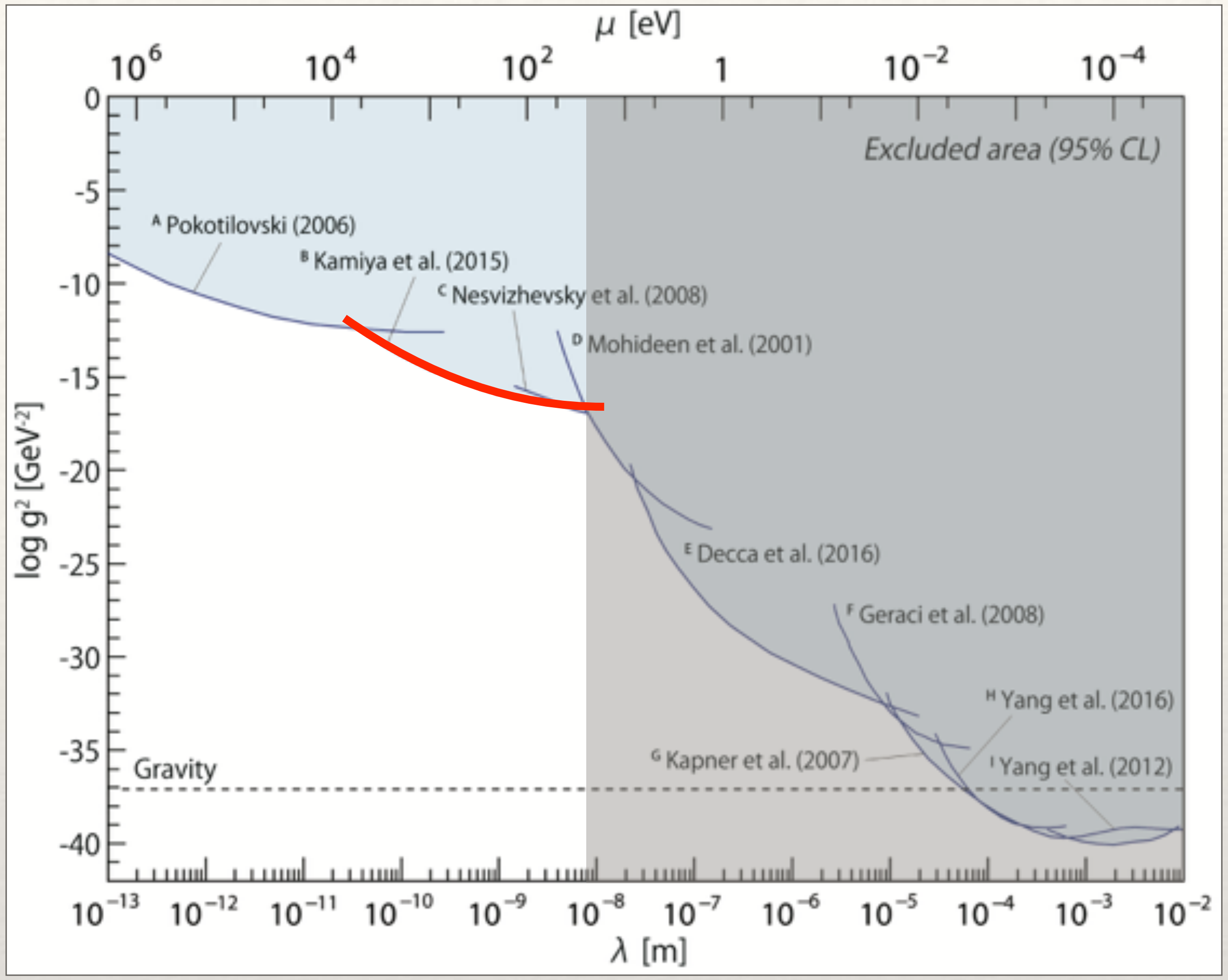
interaction range : $\lambda \equiv \frac{1}{\mu}$



1.1 Current limits

- $\lambda < 10$ nm
  neutron scattering
  - e.g. Kamiya et al. (2015)





- Precise measurement of scattering angle distribution
- Slow neutron beam (λ=5Å, E~3meV)
- 2 atm Xenon

2. Scattering law

incoming neutron momentum **k**

N atoms (tergets)

outgoing neutron momentum **k′**

$q = |\mathbf{k} - \mathbf{k}'|$
$\omega = (k^2 - k'^2)/2m$

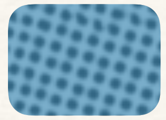
- differential cross section :

coherent / incoherent scattering length

$$\frac{d^2\sigma}{d\Omega d\omega} = N\frac{k'}{k}\left[b_{\mathrm{coh}}^2(q)S_c(q,\omega) + b_i^2 S_i(q,\omega)\right]$$

2. Scattering law

incoming neutron momentum **k** → N atoms (tergets)



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dynamic structure factor

$$S_c(q,\omega;T,\rho) = \frac{1}{2\pi N}\int_{-\infty}^{\infty} dte^{-i\omega t}\sum_{j,j'}\left\langle e^{-i\boldsymbol{q}\cdot\boldsymbol{R}_j(0)}e^{i\boldsymbol{q}\cdot\boldsymbol{R}_{j'}(t)}\right\rangle$$

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j : indices of atoms

$\mathbf{R}_j$: the position of j th atom

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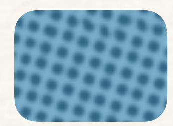

outgoing neutron momentum **k′**

q= | **k** - **k′** |
ω=(k²-k′²)/2m

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$\mathbf{R}_j$: the position of j th atom

static structure factor

$$\frac{d\sigma}{d\Omega} = N\frac{k'}{k}\left[b_{\rm coh}^2(q)S(q) + b_i^2\right]$$

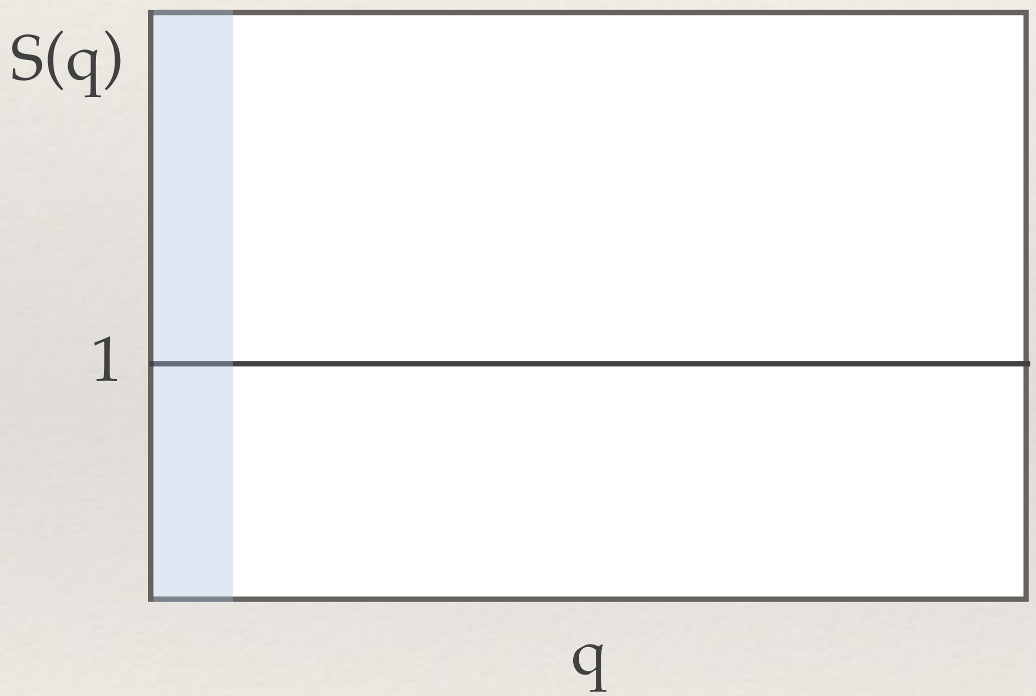
"sum rule"

$$S_0 \equiv S(0) = \rho\kappa_T k_B T = \left(\frac{\partial\rho}{\partial P}\right)_T k_B T$$

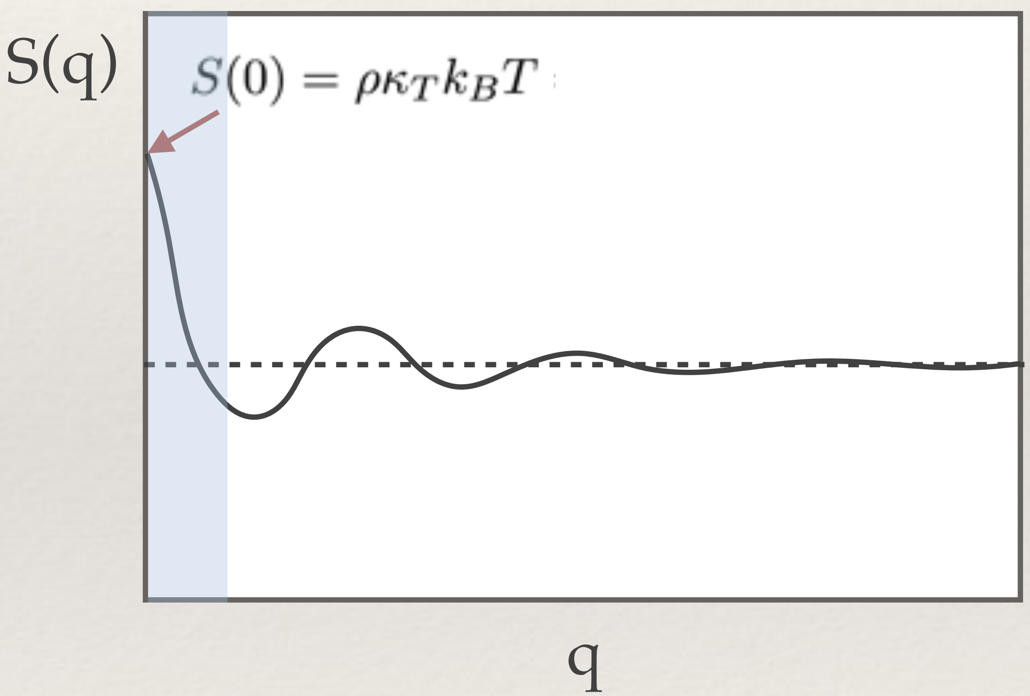
2. Scattering law

$$\frac{d\sigma}{d\Omega} = N\frac{k'}{k}\left[b_{\rm coh}^2(q)S(q) + b_i^2\right]$$

for ideal gas (illustration)



for dense gas (illustration)



2.1 coherent scattering length

- General expression (leading term, for cold neutron) :

$$b_{\mathrm{coh}}(q) \approx b_c - b_e Z[1 - f(q)]$$

for Kr

$$b_c = b_{N_c} + b_{N_p}$$

- b_{N_p}: the neutron electric polarization
- b_{N_c}: the strong interaction

~ 8.0 fm

$$b_e = b_F + b_I$$

- b_I: intrinsic n-e scattering length, neutron charge distribution - electron — ~ -1.5×10^{-3} fm
- b_F: Foldy scattering length, neutron magnetic moment - charge density of atom — ~ -1.3×10^{-3} fm

- f(q) : the atomic form factor (the empirical form)

$$f(q) = \frac{1}{\sqrt{1 + 3(q/q_0)^2}}$$

$q_0 = 1.9\ Z^{1/3}$ [1/Å]

cf. V. F. Sears, Phys. Rep. 141, 281 (1986)

2.2 static structure factor

- interatomic potential : $\varphi(r) \propto r^{-6} \quad (r \to \infty)$

  …due to the van der Waals force

- According to the fluid structure theory :

$$\frac{1}{2\pi^2 \rho r} \int_0^\infty dq \left\{ q \frac{S(q)-1}{S(q)} \right\} \sin qr \sim -\beta\varphi(r) \propto r^{-6} \qquad \text{as } r \to \infty$$

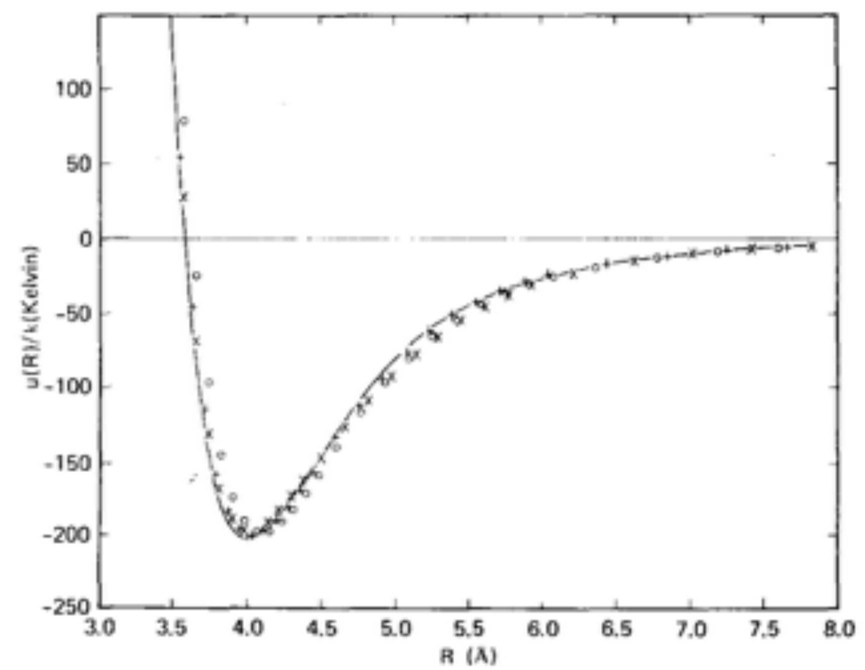


FIG. 3. Potentials for krypton. —, present potential (*K*2); o, Docken and Schafer (Ref. 8); ×, Bobetic and Barker (Ref. 15); +, Buck *et al.* (Ref. 9).

Barker et al. J. Chem. Phys., 61 (1974)

cf. L. Reatto and M. Tau, J. Phys.: Condens. Matter 4, 1 (1992)

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- From the results of "the asymptotic Fourier analysis"

$$\frac{1}{r}\int_0^\infty F(q)\sin qr dq \sim \frac{F(0)}{r^2} - \frac{F''(0)}{r^4} + \frac{F^{\mathrm{iv}}(0)}{r^6} - \dots$$

=0 =0

$$F(q) \equiv q\left\{1 - S^{-1}(q)\right\} \qquad F''(0) = -\frac{2S'(0)}{S^2(0)} = 0 \Leftrightarrow S'(0) = 0$$

$$S(q) = S_0 + S_2 q^2 + S_3 q^3 + S_4 q^4 \cdots$$

cf. L. Reatto and M. Tau, J. Phys.: Condens. Matter 4, 1 (1992)

3. Scattering length with new interactions

- The potential due to new interactions :

$$V_{\rm new}(r) = -\frac{1}{4\pi} g^2 Q_1 Q_2 \frac{e^{-\mu r}}{r} \xrightarrow{\text{Born approx.}} b_{\rm new}(q) = \frac{m_n}{2\pi} g^2 Q_1 Q_2 \frac{1}{\mu^2 + q^2}$$

- The scattering length :

$$b_{\rm coh}(q) \approx b_c - b_e Z[1 - f(q)] + b_{\rm new}(q)$$
$$= b_c \left\{ 1 + \chi_{\rm em}[1 - f(q)] + \chi_{\rm new} \frac{\mu^2}{q^2 + \mu^2} \right\}$$

$$\chi_{\rm em} \equiv -\frac{b_e}{b_c} Z \qquad \chi_{\rm new} \equiv \frac{m_n}{2\pi} g^2 Q_1 Q_2 \frac{1}{b_c \mu^2}$$

For Kr, $\chi_{\rm em} \sim 10^{-2}$ $\chi_{\rm new} \sim 10^{-3}$ $(1/\mu = 10^{-9} m,\ g^2 = 10^{-15})$

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$$b_{\mathrm{coh}}^2(q) \approx b_c^2 \left\{ 1 + 2\chi_{\mathrm{em}}[1 - f(q)] + 2\chi_{\mathrm{new}} \frac{\mu^2}{q^2 + \mu^2} \right\}$$

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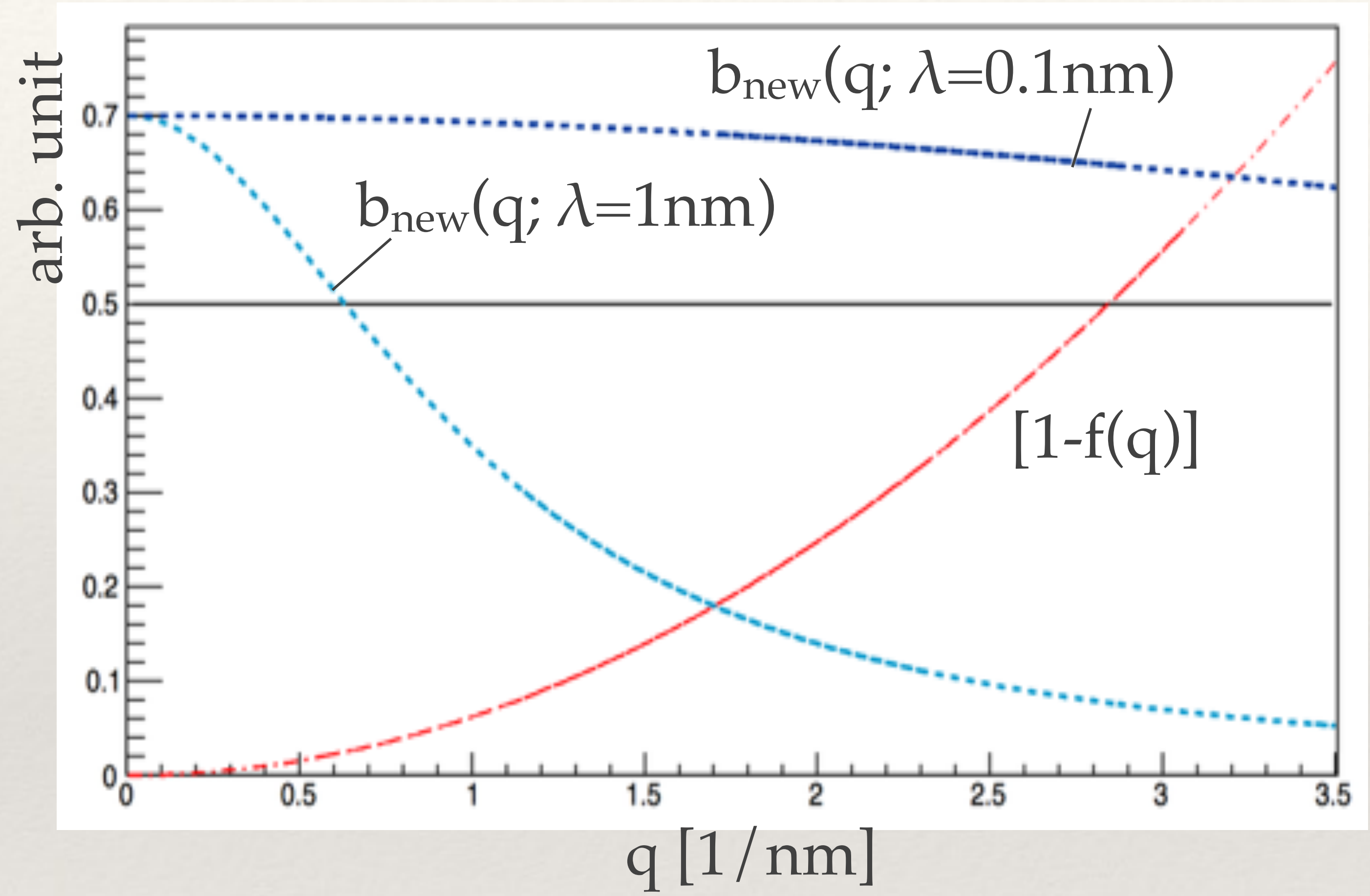


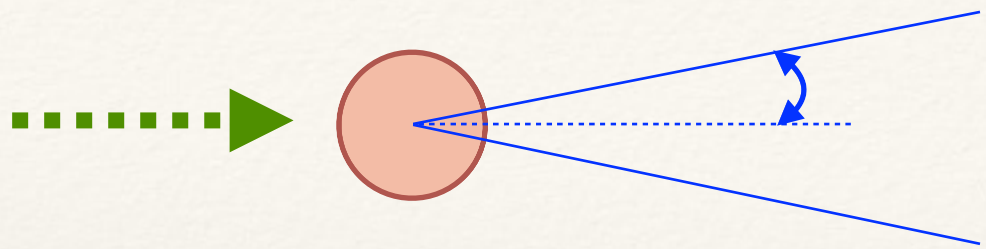
The differential cross section (coherent term) :

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm coh} = N\frac{k'}{k} b_c^2(q) S(q)$$

$$= N\frac{k'}{k} \times b_c^2 \left\{ 1 + 2\chi_{\rm em}[1 - f(q)] + 2\chi_{\rm new} \frac{\mu^2}{q^2+\mu^2} \right\} \times (S_0 + S_2 q^2 + S_3 q^3 + \cdots)$$

4. 実験条件の検討

- シミュレーション(with McStas)のセットアップ



sample
- 半径　r
- S(q) data：Teitsma et al. (1980)

barn	$4\pi b_c^2$	$4\pi b_i^2$	σ_{abs}
nKr	7.67	0.01	25
86Kr	8.2	0	0.003

beam
- 波長 5Å
- 直径 16mm
- 発散角 3mrad

detector
- 散乱角： 167 mrad

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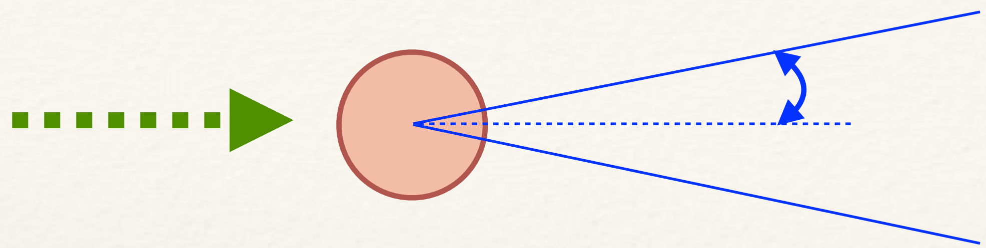

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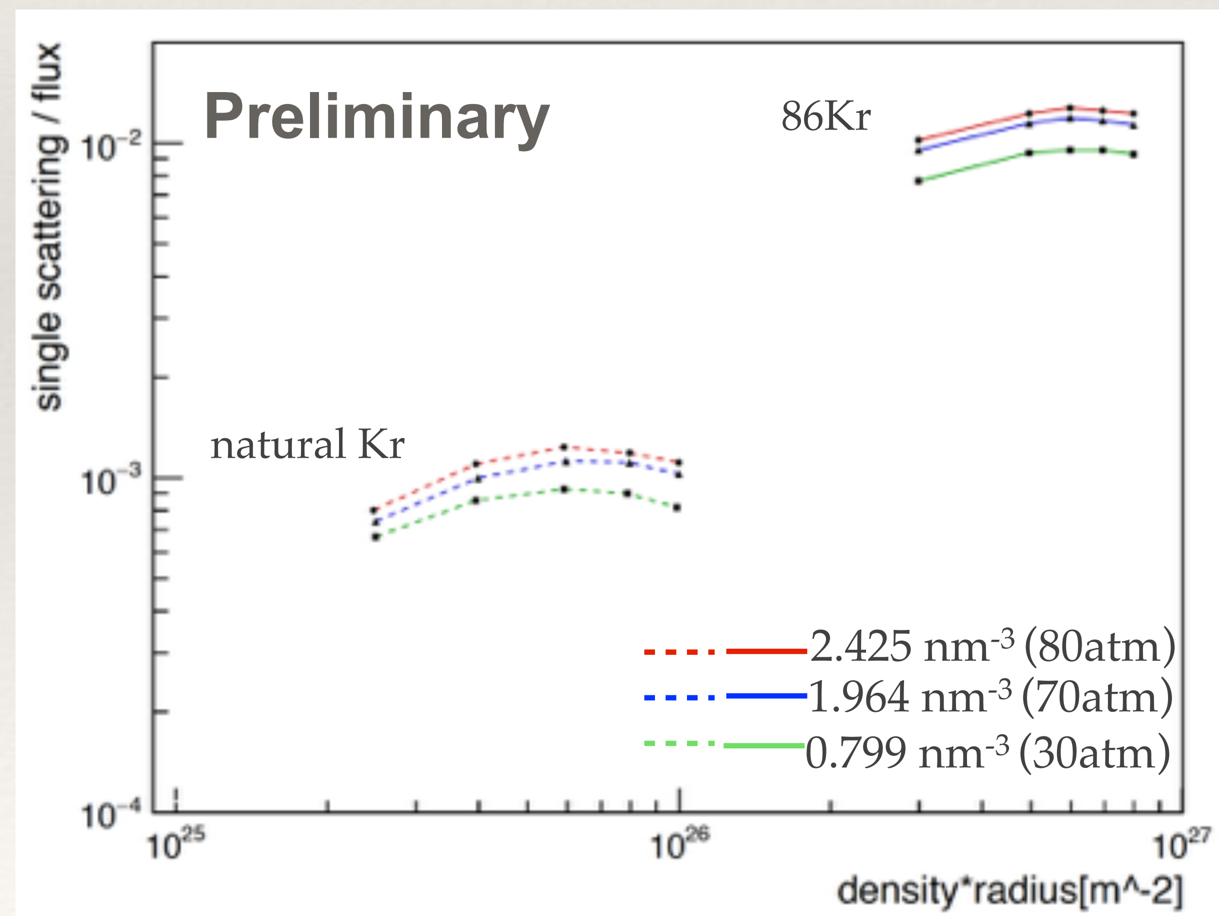
- 自己遮蔽により散乱強度は頭打ち
- 最も強くなる条件

	density*radius[m^-2]
nKr	6.2×10^{25}
86Kr	6.6×10^{26}



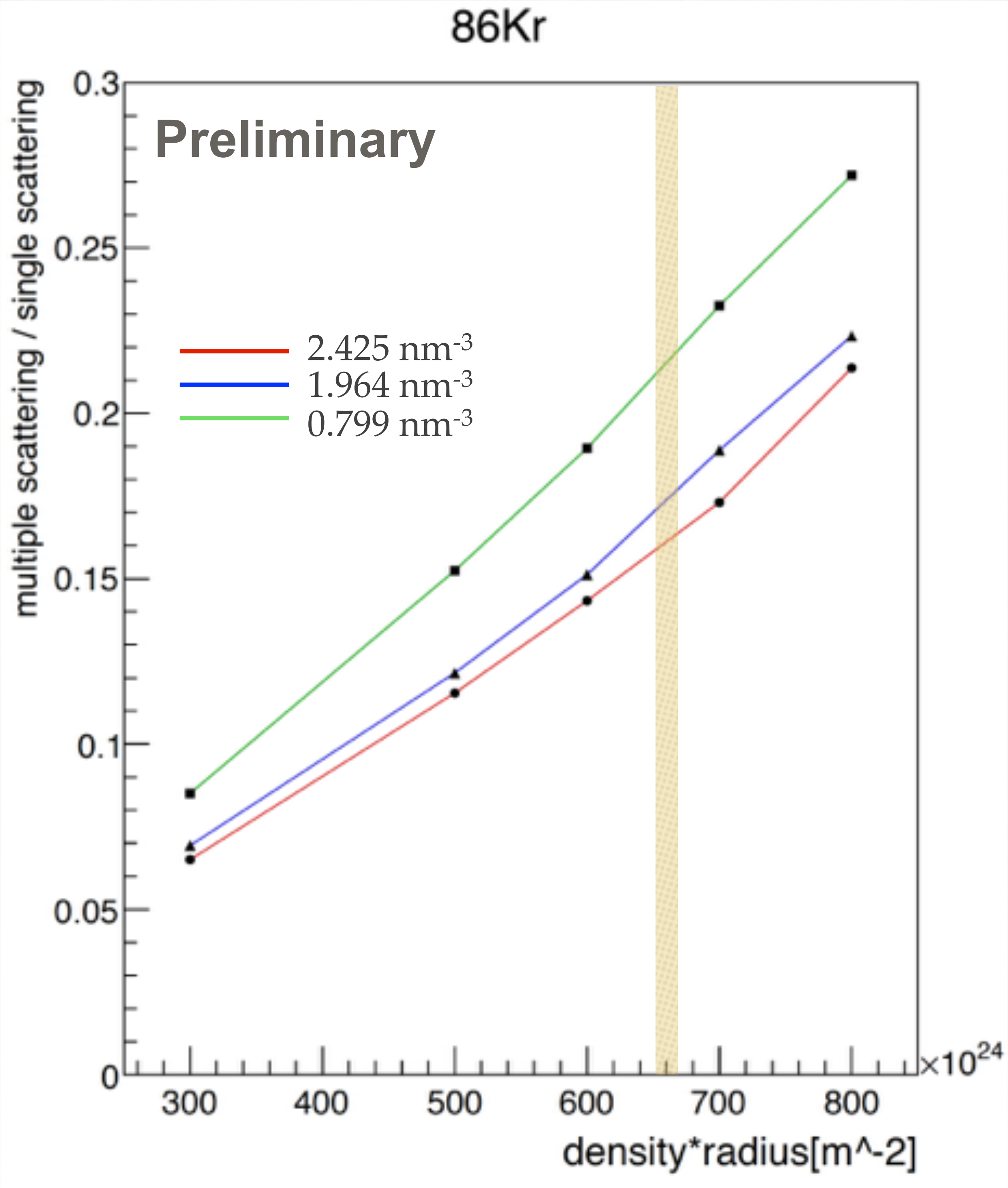
4. 実験条件の検討

- 1回散乱と多重散乱の比
  - 86Krの1回散乱が最大

    →多重散乱＞10%

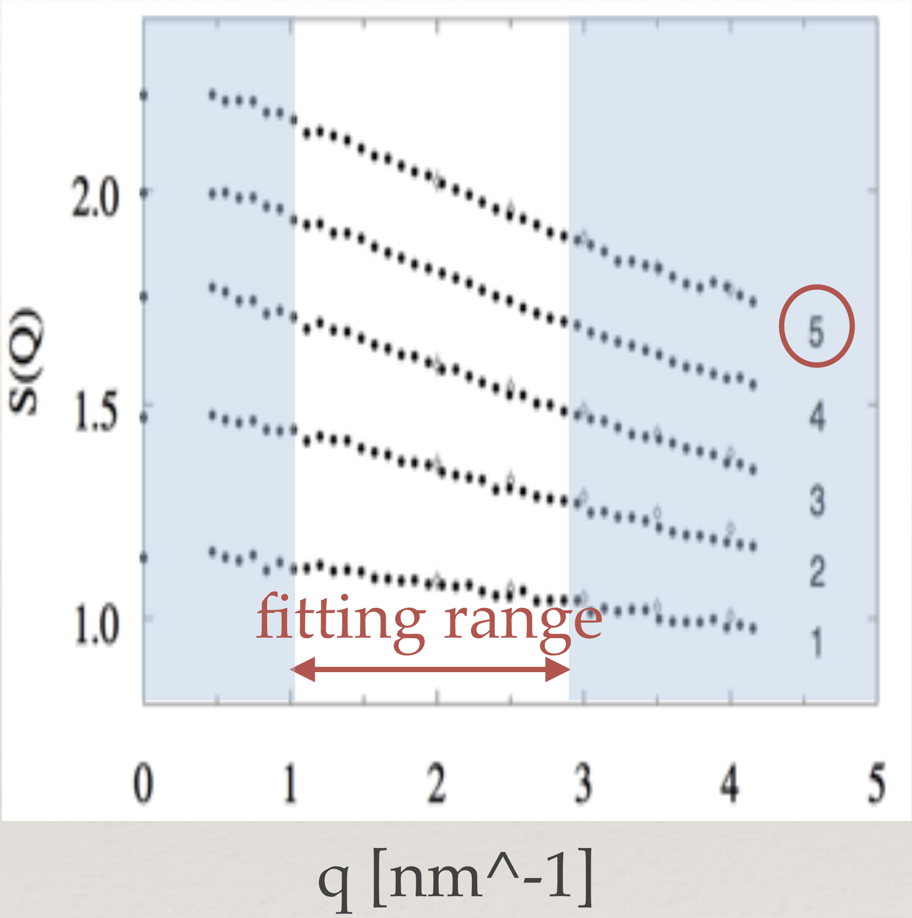
- 仮に多重散乱＜1%未満

  とすると **86Krにするご利益はない**



5. Analysis

- Benmore et al. J. Phys.: Condens. Matter, **11**, 3091(1999)
- Experimental conditions
  - **86Kr** gas fluid, $\sigma_c = 4\pi b_c^2 = 8.2 \pm 0.5$ [barn]
  - T=297.6±0.5 K
  - ϱ=0.804, 1.522, 1.984, 2.231, 2.431 nm^{-3} (~80atm)
  - the statistical error : 0.5%



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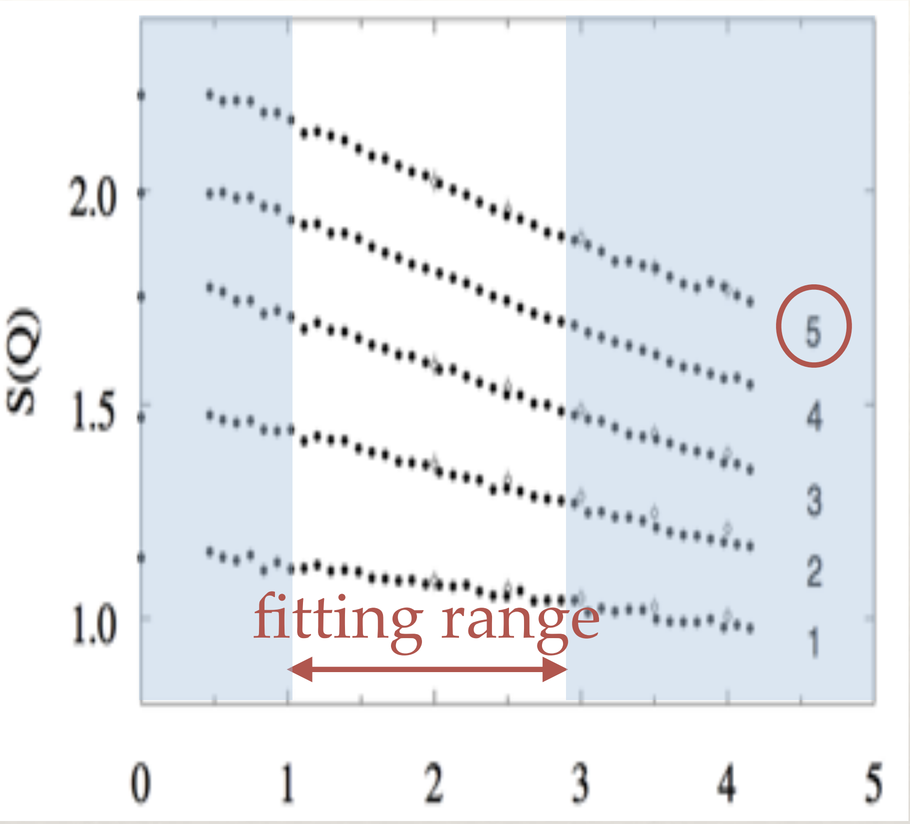


q [nm^-1]

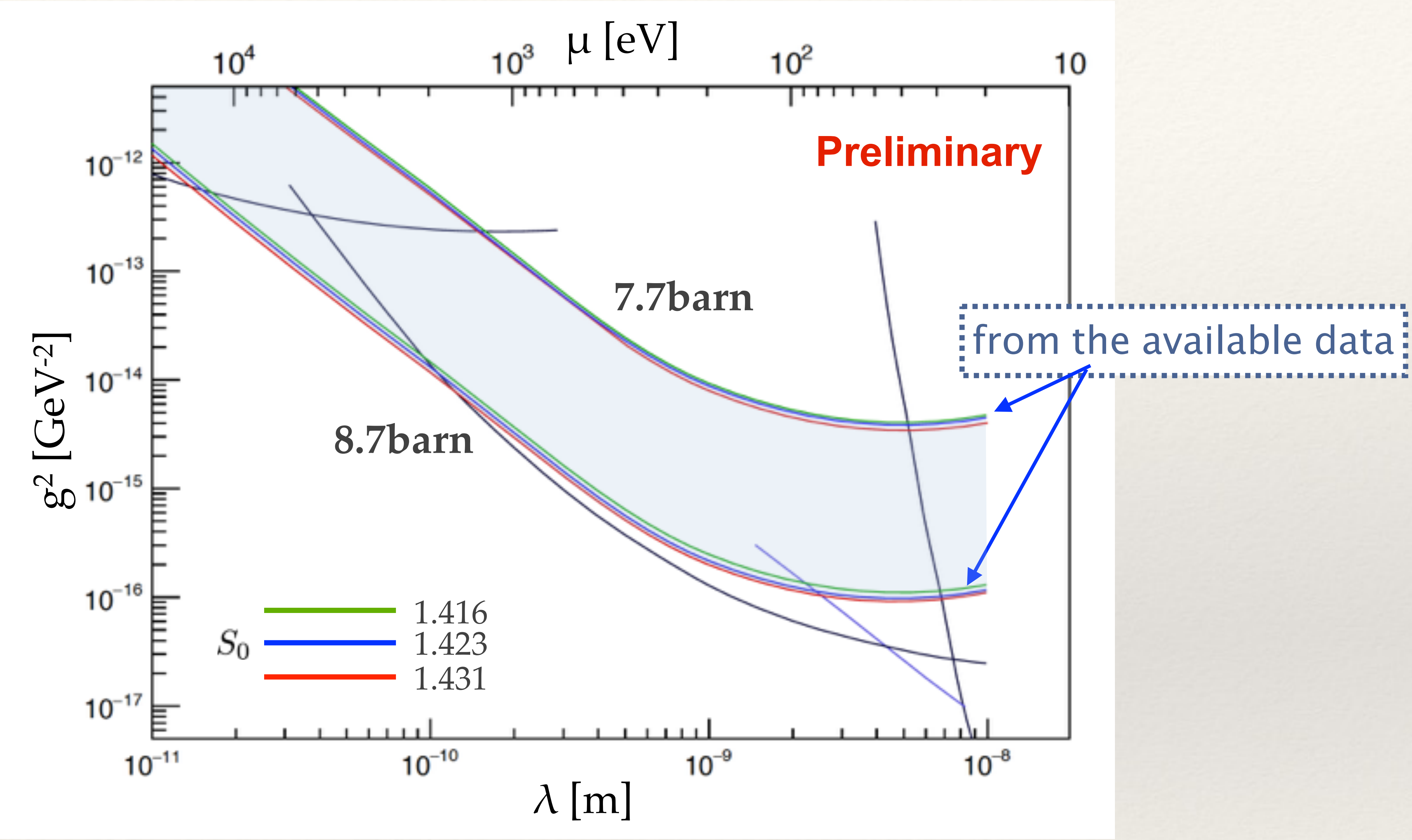
- The fitting function

$$\left\{1 + 2\chi_{\rm em}[1 - f(q)] + 2\chi_{\rm new}\frac{\mu^2}{q^2+\mu^2}\right\} \times (S_0 + S_2 q^2 + S_3 q^3)$$

$$S_0 = 1.423 \pm 0.008$$

  - The fitting range : $1 < q < 2.8\ nm^{-1}$
    - lower limit : to ignore the retardation effect
    - upper limit : to avoid contributions of higher order terms

5. Analysis



μ [eV]
10^4
10^3
10^2
10
Preliminary
7.7barn
8.7barn
from the available data
g^2 [GeV^-2]
10^-12
10^-13
10^-14
10^-15
10^-16
10^-17
S_0
1.416
1.423
1.431
10^-11
10^-10
10^-9
10^-8
λ [m]

6. Summary

- **濃縮流体**を用いた、質量に結合する未知短距離力の**探索手法を開発**
- natural Kr, 86Krサンプルに対する**散乱強度および多重散乱を評価**
    - 86Krを用いるメリットは無い(多重散乱/1回散乱<1%)
- 86Kr小角散乱実験のデータに本手法を適用
    - 断面積の不定性、S_0の不定性を考慮し、**到達感度を評価**

今後

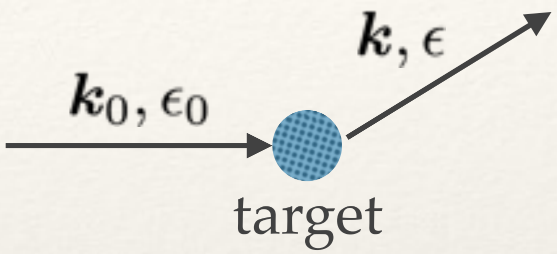
- 他の不定性(純度, 温度変化, 散乱位置等)を考慮し解析

Multiple Scattering

- The Double Differential Cross Section **in the small-sample limit**

$$\frac{d^2\sigma}{d\Omega d\epsilon} = N\frac{\sigma_s}{4\pi}\frac{k}{k_0}S(Q,\omega)$$

  - N : the number of target atoms
  - σ_s : the scattering cross section
  - $S(Q,\omega)$: the dynamical structure factor
    (the Van Hove response func.)



$$\boldsymbol{Q} = \boldsymbol{k}_0 - \boldsymbol{k}$$
$$\omega = \epsilon_0 - \epsilon$$

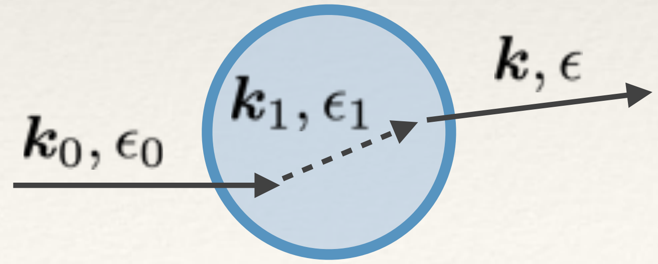
- The Double Differential Cross Section (the general form)

$$\frac{d^2\sigma}{d\Omega d\epsilon} = N\frac{\sigma_s}{4\pi}\frac{k}{k_0}\sum_{j=0}^{\infty} s_j(\boldsymbol{k}_0,\boldsymbol{k})$$

  - $s_j(\boldsymbol{k}_0,\boldsymbol{k})$: the contribution from neutrons which have been scattered j times
    - single $s_1(\boldsymbol{k}_0,\boldsymbol{k}) = S(\boldsymbol{Q},\omega)H_1(\boldsymbol{k}_0,\boldsymbol{k})$
    - double $s_2(\boldsymbol{k}_0,\boldsymbol{k}) = \frac{n\sigma_s}{4\pi}\int d\Omega_1 d\epsilon_1 S(\boldsymbol{Q}_1,\omega_1)S(\boldsymbol{Q}_2,\omega_2)H_2(\boldsymbol{k}_0,\boldsymbol{k}_1,\boldsymbol{k})$

    …

* $H_j(\boldsymbol{k}_0,\cdots,\boldsymbol{k})$: the transmission factor



$$\boldsymbol{Q}_1 = \boldsymbol{k}_0 - \boldsymbol{k}_1$$
$$\boldsymbol{Q}_2 = \boldsymbol{k}_1 - \boldsymbol{k}$$
$$\omega_1 = \epsilon_0 - \epsilon_1$$
$$\omega_2 = \epsilon_1 - \epsilon$$

- We require a knowledge of $S(Q,\omega)$
  →multiple scat. corrections are important